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DRIFT OF A RAREFIED GAS IN A PLANAR CHANNEL UNDER THE ACTION OF MONOCHROMATIC RADIATION

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UDC 533.6.022.8:535.375.5

A theoretical study is performed of light-induced drift of a rarefied gas in a planar channel. The problem is solved using linearized kinetic equations for a two-level particle model in the "weak field" approximation. Analytical expressions are obtained for the drift, averaged over channel section in almost free-molecular (Kn $\gg$ 1) and viscous with slippage (Kn $\ll$ 1) flow regimes. Various mechanisms for this drift are analyzed. Numerical estimates of drift velocity are presented.

The phenomenon of light-induced drift involves [1] particles absorbing radiation in the form of a travelling wave while located in a mixture with a buffer gas, and taking on a directed motion (drift). The drift may occur in the direction of radiation propagation or opposite thereto. It has been established that this drift phenomenon is realized in cases typical of nonlinear optics and spectroscopy problems, and is inherent to several classes of particles: atoms, molecules, and ions.

The great majority of studies of light-induced drift have investigated the phenomenon in an infinite gas. At the same time it is clear that light-induced drift is also possible in the case where the role of the buffer gas is played by an interphase surface, which reflects excited and unexcited particles in different manners. The factor of gas—surface interaction becomes one of the dominant ones in drift motion in capillary-porous media.

In [2] gas drift was studied in a vessel, the dimensions of which were large in comparison to the atomic free path length. The model used in [2] of strong collisions with a single Maxwell collision frequency can be considered only as a first approximation for description of light-induced drift. In particular, it does not consider the collision mechanism of light-induced drift development related to difference in the frequency of collisions of excited and unexcited atoms among themselves [3, 4]. Moreover, to calculate the velocity of light-induced slippage, [2] considered the spatially homogeneous case, although it is precisely in the Knudsen layer that the macroparameters (and thus, the distribution function) experience their greatest changes.

In [5] light-induced drift was studied in a planar channel on the basis of specific model equations [6], according to which each act of interatomic collision leads to extinction of an excitation, and the saturation parameter is independent of particle velocity in the resonance region. The latter is possible only in the case of detuning of the radiation frequency from resonance Ω and a Doppler shift kv. In [6] the accommodation mechanism of light-induced drift was not considered.

In the present study light-induced drift in a plannar channel will be described by second order model kinetic equations, which in contrast to the strong collision model include as macroparameters the gas velocity and the stress tensor, as well as considering three types of interparticle interactions. The problem is solved by the variation method, which permits achievement of sufficiently precise results over the entire Knudsen number (Kn) range, and in addition leads to simple analytical expressions for the light-induced drift in the two limiting cases Kn $\ll$ 1 and Kn $\gg$ 1. This permits a clear analysis of the contribution of both the collision and the accommodation mechanisms to gas drift at various Kn.

A. M. Gor'kii Ural State University, Sverdlovsk. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 60, No. 6, pp. 1015-1021, June, 1991. Original article submitted July 17, 1990.

 0022-0841/91/6006-0788$12.50

We will consider steady state motion of a monatomic gas between infinite parallel plates $\tilde{x} = \pm d/2$, caused by resonant interaction of the gas with light directed along the z-axis. Collisions of atoms with each other and with the boundary surface are assumed elastic. The absorbing particles have two energy levels (n, m are the ground and excited states), i.e., interaction of the radiation with the monatomic gas leads to formation of a binary mixture of atoms having identical masses, but differing collision cross sections. The state of such a gas mixture can be described by distribution functions f_i, satisfying the kinetic equations [7]:

$$\begin{gathered} v_x \frac{\partial f_n}{\partial \tilde{x}} = \frac{\Gamma_m \varkappa(\mathbf{v})}{2}(f_m - f_n) + \Gamma_m f_m + S_n, \\ v_x \frac{\partial f_m}{\partial \tilde{x}} = \frac{\Gamma_m \varkappa(\mathbf{v})}{2}(f_n - f_m) - \Gamma_m f_m + S_m, \\ \varkappa(\mathbf{v}) = \frac{4|G|^2\Gamma}{\Gamma_m[\Gamma^2 + (\Omega - \mathbf{kv})^2]}, \quad G = \frac{Ed_{mn}}{2\hbar}, \quad \Omega = \omega - \omega_{mn}. \end{gathered} \tag{1}$$

The absorption saturation parameter $\varkappa(\mathbf{v})$ characterizes the probability of individual transitions and is a function of particle velocity in the general case. The parameter $\varkappa$ is proportional to radiation intensity and consequently depends on coordinate z due to absorption of radiation by the gas. However in the case of an optically thin medium or at relatively small distances the drift velocity can be considered independent of z. Moreover, the radiation is assumed homogeneous over channel height.

If the probability of of individual transitions is small (weak field, $\varkappa \ll 1$), then the state of each gas component is only slightly disturbed and the distribution functions can be written in the form of slight deviations from the Maxwell distributions f_{i0}:

$$\begin{gathered} f_i = f_{i0}[1 + h_i(x, \mathbf{v})], \quad i = n, m, \\ f_{i0} = n_{i0}\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right), \end{gathered} \tag{2}$$

where n_{i0} is the equilibrium numerical density of component i, T is the equilibrium gas temperature.

We will also assume the frequency of collisions between atoms is much greater than the radiation attenuation constant:

$$\Gamma_m^{(i)} = \frac{\Gamma_m}{(\gamma_{ii} + \gamma_{ij})} \ll 1. \tag{3}$$

This condition imposes certain limitations on the minimum possible gas pressure.

The linearized kinetic equations (1) with second order collision integrals in MacCormack form [8] take on the following form:

$$\begin{gathered} c_x \frac{\partial h_i}{\partial x} = \delta_i \left\{ \frac{\Gamma_m^{(i)} \varkappa}{2}\left(\frac{n_j - n_i}{n_i}\right) + 2c_z[u_i - \varphi_{ij}^{(1)}(u_i - u_j)] + \right. \\ \left. + 4c_x c_z[(1 - \varphi_{ii}^{(3)} + \varphi_{ii}^{(4)} - \varphi_{ij}^{(3)})\pi_{ixz} + \varphi_{ij}^{(4)}\pi_{jxz}] - h_i \right\}, \quad i = n, m, \ i \neq j, \end{gathered} \tag{4}$$

where

$$x = \frac{\tilde{x}}{d}; \quad \delta_i = \frac{d(\gamma_{ii} + \gamma_{ij})}{\bar{v}}; \quad c_i = \frac{v_i}{\bar{v}}; \quad \bar{v} = \left(\frac{2k_B T}{m}\right)^{1/2};$$

$$\varphi_{ij}^{(n)} = \frac{\nu_{ij}^{(n)}}{(\gamma_{ii} + \gamma_{ij})}\ ;\ \varphi_{ii}^{(n)} = \frac{\nu_{ii}^{(n)}}{(\gamma_{ii} + \gamma_{ij})}\ ;$$

$$u_i = \frac{U_i}{\bar{v}} = \pi^{-3/2} \int c_z h_i \exp(-c^2)\, dc;$$

$$\pi_{ixz} = \frac{P_{ixz}}{2p_i} = \pi^{-3/2} \int c_x c_z h_i \exp(-c^2)\, dc; \tag{5}$$

δ_i is the gas rarefaction parameter, inversely proportional to Knudsen number Kn, u_i, π_{ixz}, p_i characterize the macroscopic velocity, stress tensor, and pressure for mixture component i, the quantities $\nu_{ij}^{(n)}$, dependent on the concentration and Chapman–Cowling integrals, are defined in [8].

As boundary conditions we use the model of mirror-diffuse reflection, according to which the fraction $(1-\varepsilon_i)$ of atoms of type i is reflected specularly, while the fraction ε_i is reflected diffusely with Maxwellian velocity distribution but a numerical density n_{ir} differing little from equilibrium: $n_{ir} = n_{i0}(1 + \nu_{ir})$. The reflected molecules have the temperature of the wall. Then with consideration of linearization of Eq. (2) the boundary conditions for the perturbation functions take on the form

$$h_i\left(x = -\frac{1}{2},\ c_x > 0\right) = \varepsilon_i \nu_{ir} + (1 - \varepsilon_i)\, h_i\left(x = -\frac{1}{2},\ c_x < 0\right). \tag{6}$$

We then have the symmetry conditions

$$h_i\left(x = -\frac{1}{2},\ c_x < 0\right) = h_i\left(x = \frac{1}{2},\ c_x > 0\right),$$

$$h_i\left(x = \frac{1}{2},\ c_x < 0\right) = h_i\left(x = -\frac{1}{2},\ c_x > 0\right). \tag{7}$$

The coefficients ε_i characterizing the fraction of diffusely scattered excited and nonexcited atoms differ in the general case ($\varepsilon_m \neq \varepsilon_n$).

Writing Eq. (4) in integral form with consideration of boundary conditions (6), (7) and substituting the expressions obtained for the perturbation functions in Eq. (5), we obtain a system of integral–moment equations for the macroscopic velocity and stress tensor for mixture component i:

$$u_i = \frac{\delta_i}{\sqrt{\pi}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \{u_i K_1 - \varphi_{ij}^{(1)}(u_i - u_j) K_1 + 2K_2 [(1 - \varphi_{ii}^{(3)} + \varphi_{ii}^{(4)} - \varphi_{ij}^{(3)})\, \pi_{ixz} + \varphi_{ij}^{(4)} \pi_{jxz}] + A_i \langle \varkappa \rangle K_1\}\, ds, \tag{8}$$

$$\pi_{ixz} = \frac{\delta_i}{\sqrt{\pi}} \int_{-\frac{1}{2}}^{+\frac{1}{2}} \{u_i K_2 - \varphi_{ij}^{(1)}(u_i - u_j) K_2 + 2K_3 [(1 - \varphi_{ii}^{(3)} + \varphi_{ii}^{(4)} - \varphi_{ij}^{(3)})\, \pi_{ixz} + \varphi_{ij}^{(4)} \pi_{jxz}] + A_i \langle \varkappa \rangle K_2\}\, ds, \tag{9}$$

$$i,\ j = n,\ m;\quad i \neq j,$$

where

$$K_1 = J_{-1}(t) + \sum_{l=1}^{\infty} (1 - \varepsilon_i)^l [J_{-1}(t_1) + J_{-1}(t_2)];$$

$$K_2 = J_0(t)\operatorname{sgn}(x-s) + \sum_{l=1}^{\infty}(1-\varepsilon_i)^l [J_0(t_1) - J_0(t_2)];$$

$$K_3 = J_1(t) + \sum_{l=1}^{\infty}(1-\varepsilon_i)^l [J_1(t_1) + J_1(t_2)];$$

$$t = \delta_i |x-s|; \quad t_1 = \delta_i (x-s+l); \quad t_2 = \delta_i (s-x+l);$$

$$A_i = \frac{\Gamma_m^{(i)}}{2\sqrt{\pi}}\left(\frac{n_j - n_i}{n_i}\right); \quad \langle \varkappa \rangle = \int_{-\infty}^{+\infty} c_z \varkappa \exp(-c_z^2)\, dc_z.$$

In Eqs. (8), (9) the quantities $J_n(x)$ are transcendental functions of the form [9]:

$$J_n(x) = \int_0^{\infty} c^n \exp\left(-c^2 - \frac{x}{c}\right) dc.$$

To calculate the quantity $\langle \varkappa \rangle$ it will be convenient to use the plasma function, numerical values of which are tabulated in [10]. For the two limiting cases of homogeneous ($\Gamma \gg k\bar{v}$) and inhomogeneous ($\Gamma \ll k\bar{v}$) broadening the expression for $\langle \varkappa \rangle$ has the following form:

$$\langle \varkappa \rangle = \begin{cases} \dfrac{4\pi\Omega}{\Gamma_m}\left(\dfrac{|G|}{k\bar{v}}\right)^2, & \Gamma \ll k\bar{v}, \\ \dfrac{4\sqrt{\pi}\Omega(k\bar{v})\Gamma}{\Gamma_m}\left(\dfrac{|G|}{\Omega^2+\Gamma^2}\right)^2, & \Gamma \gg k\bar{v}, \end{cases} \tag{10}$$

$k = 2\pi/\lambda$ is the wave number, and λ is the wavelength.

Equations (8), (9) define the local values of the macroscopic quantities. But the quantity of practical interest is the numerical flux I, averaged over the channel section and defining the number of atoms passing through the channel section per unit time:

$$I = n\langle U \rangle = I_n + I_m = \bar{v} \int_{-\frac{1}{2}}^{+\frac{1}{2}} (n_n u_n + n_m u_m)\, dx. \tag{11}$$

To determine the numerical flux of Eq. (11) it is necessary to solve the system of integral–moment equations (8), (9). These are Fredholm equations of the second sort. Consequently they can be solved by the Buvnov-Galerkin method [11]. In this case the choice of test functions for the macroscopic quantities is important. It must be kept in mind that the macroscopic quantity profiles in the almost free-molecular regime are described precisely by the free terms of integral equations (8), (9). Therefore, in order that the solution contain the other limiting case (Kn $\ll$ 1), the test function form should be chosen on the basis of the behavior of the macroparameters in the regime with slippage. Considering the problem symmetry, the unknown functions u_i, π_{ixz} can be approximated in the following manner:

$$u_i = a_{1i} + a_{2i}x^2, \quad \pi_{ixz} = a_{3i}x, \quad i = n, m. \tag{12}$$

The test functions of Eq. (12) correspond in form to solutions of the Navier–Stokes equations.

To determine the constants a_{1i}, a_{2i}, a_{3i} the test functions of Eq. (12) must be substituted in system (8)-(9), while requiring orthogonality of the expressions obtained to the base functions, with the orthogonality condition for arbitrary functions f and g having the form:

$$(f, g) = \int_{-\frac{1}{2}}^{+\frac{1}{2}} f(x)\, g(x)\, dx = 0.$$

Thus, the quantities a_{1i}, a_{2i}, a_{3i} are found by solution of a system of six linear algebraic equations. Having defined those values, we can obtain an expression for the numerical light-induced drift flux. The final expression is cumbersome in form, and so will not be presented. For simplification we can use the realistic approximation $n_m/n_n \ll 1$, which is valid for weak radiation intensity. At intermediate Kundsen numbers a numerical calculation is necessary; analytical expressions for the numerical flux can be obtained for the two limiting cases: the almost free-molecular regime ($\delta_n \ll 1$)

$$I = n_n \langle U \rangle = -\frac{n_n \Gamma_m d \langle \varkappa \rangle}{\pi} \left(\frac{\varepsilon_n - \varepsilon_m}{\varepsilon_n \varepsilon_m} \right) \ln \delta_n + \ldots; \tag{13}$$

and the regime with slippage ($\delta_n \gg 1$)

$$I = \frac{n_n d \Gamma_m \langle \varkappa \rangle}{2\sqrt{\pi}\theta\varphi_{mn}^{(1)}} \left[\frac{\varepsilon_n - \varepsilon_m}{\varepsilon_n} + \right.$$
$$\left. + \frac{\varepsilon_m}{2} \left(1 + \frac{\varphi_{nn}^{(4)} - \varphi_{nn}^{(3)}}{\theta\varphi_{mn}^{(3)}} - \frac{\varphi_{mn}^{(4)}}{\varphi_{mn}^{(3)}} \right) \right] \frac{1}{\delta_n} + \ldots, \quad \theta = \frac{\gamma_{mn}}{\gamma_{nn}}. \tag{14}$$

Asymptotic analysis reveals that in the almost free-molecular flow regime as $\delta_n \to 0$ the absolute value of the light-induced drift increases logarithmically. This can be explained by the degeneration of the channel geometry (infinite plate dimensions). In the regime with slippage as $\delta_n \to \infty$ the light-induced drift vanishes as δ_n^{-1}.

The direction of atomic drift depends on the ratio of the particle dimensions in the excited and ground states, the character of the interaction with the boundary surface, and the sign of Ω, the detuning of the radiation frequency from the center of the absorption line.

In the almost free-molecular regime ($\delta_n \ll 1$), Eq. (13), the drift is produced only by the accommodation mechanism, since collisions between atoms are insignificant in this regime. If $\Delta\varepsilon = \varepsilon_n - \varepsilon_m > 0$, then for $\Omega > 0$ the gas drifts in the direction of the radiation, and for $\Omega < 0$, opposite the radiation.

In the regime with slippage ($\delta_n \gg 1$) Eq. (14) includes a new mechanism producing gas drift, the collision mechanism. If the atoms interact with the interphase boundary identically ($\varepsilon_i = \varepsilon$), then for different collision frequencies of excited and nonexcited particles among themselves ($\theta \neq 1$, $\sigma_{mn} \neq \sigma_{nn}$) light-induced drift develops. This mechanism appears only in a limited gas volume. In an infinite gas the difference in particle collision frequencies does not lead to drift of the gas as a whole in view of the law of conservation of momentum. In a finite volume oppositely directly fluxes of excited and nonexcited particles correspond to different Knudsen numbers (since the collision sections are different), and thus, have different magnitudes. In this case, the light-induced drift is caused by the difference in fluxes, and its direction coincides with the direction of the nonexcited particle flux. The collision mechanism was first noted in [3, 4, 12] and confirmed in [5].

If $\theta = 1$ ($\sigma_{mn} = \sigma_{nn}$), but $\Delta\varepsilon = \varepsilon_n - \varepsilon_m \neq 0$, the accommodation mechanism enters into the drift, in a manner similar to the case with a buffer in an infinite gas. In this case the light-induced drift is caused by the different accommodation of the particles with the surface, which agrees qualitatively with [2]. The sign of the drift is determined by the ratio of the coefficients ε_n, ε_m and the sign of the detuning Ω. If $\Delta\varepsilon = \varepsilon_n - \varepsilon_m > 0$, then for $\Omega > 0$ the drift is directed along the radiation; if $\Delta\varepsilon < 0$, the direction and magnitude of the numerical flux are determined by the contribution of each of the mechanisms: accommodation and collision.

At exact resonance ($\Omega = 0$) gas drift is absent in any flow regime. In this case the spectral line of the set of particles will as before be a symmetrical contour with center on the quantum transition frequency ω_{mn}.

We will note that the expression for the numerical flux at $\delta_n \gg 1$, Eq. (14), is universal (it does not depend on the form of the interphase surface) and can serve as a slippage boundary condition for macroscopic laser gas dynamics equations.

We will now estimate the velocity of light-induced drift, averaged over the channel section for typical atomic characteristics. To do this we use the hard sphere molecule model. Then the expression for drift velocity in the regime with slippage ($\delta_n \gg 1$) for ($\Delta\varepsilon$, $\Delta\sigma$) $\ll 1$ for the case of inhomogeneous broadening ($\Gamma \ll k\bar{v}$) has the following form:

$$\langle U \rangle = 2\sqrt{\pi}\,\bar{v}\left(\frac{\Gamma_m}{\nu_{mn}^{(1)}}\right)\frac{\Omega}{\Gamma_m}\left(\frac{|G|}{k\bar{v}}\right)^2\left[\frac{\varepsilon_n-\varepsilon_m}{\varepsilon_n}+\frac{3}{8}\varepsilon_n\frac{\Delta\sigma}{\sigma_{nn}}\right]. \qquad (15)$$

If

$$\frac{\Delta\varepsilon}{\varepsilon_n}\sim 0{,}1;\quad \frac{\Delta\sigma}{\sigma_{nn}}=\frac{\sigma_{mn}-\sigma_{nn}}{\sigma_{nn}}\sim 0{,}1;\quad \frac{\Gamma_m}{\nu_{mn}^{(1)}}\sim 0{,}1;\quad \Omega\sim 10^9\ \text{Hz};$$

$$\Gamma_m\sim 10^7\ \text{Hz};\quad |G|\sim 10^8\ \text{Hz}\ (I=1\ \text{W/cm}^2);\quad \bar{v}\sim 10^3\ \text{m/sec};\quad \varepsilon_n\sim 0{,}95,$$

then <U> $\approx$ 0.5 m/sec. If $\Delta\varepsilon = 0$, then <U> $\approx$ 0.13 m/sec, i.e., the collision mechanism produces a contribution of the order of 25% to the light-induced drift.

In conclusion we will note that expression for drift velocity at ($\delta_n \gg 1$) can be used to determine gas–surface interaction parameters and transport sections for excited and nonexcited atoms.

NOTATION

Γ_m, radiation decay constant for level m; Γ, homogeneous absorption line width; E, radiation electric field amplitude; d_{mn}, dipole moment matrix element; $\hbar$, Planck's constant; S_i, collision integral; k, incident wave vector; ω, monochromatic radiation frequency; m, particle mass; k_B, Boltzmann's constant; γ_{ij}, frequency of collisions between type i and j atoms; $\bar{v}$, most probable particle velocity; I, radiation intensity; σ_{ij}, collision cross sections of type i and j particles.

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